Outage Performance of a Multiuser Distributed Antenna System in Underwater Acoustic Channels

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Abstract—The distributed antenna system (DAS) has been known for providing a large coverage and throughput in terrestrial radio environments. In this work, we investigate the benefits of DAS relative to the centralized antenna system (CAS) in underwater acoustic (UWA) environments. In particular, we analyze the outage performance in the wideband uplink transmissions. Based on theoretical derivation and numerical results, we show that at a medium transmission SNR regime, DAS outperforms CAS in both single-user and multiuser systems. To address the co-channel interference in the multiuser DAS, we suggest a multicell structure, which is shown having a lower outage error floor relative to the structure which takes the operation area as a single cell.

Index Terms—Distributed antenna system, underwater acoustic channels, outage performance

I. INTRODUCTION

The last decade has witnessed significant progress in underwater acoustic (UWA) communications, with typical advancements for both single-carrier and multicarrier techniques in e.g. [1]–[5]. As the point-to-point communication gradually becomes mature, interests start growing on the underwater acoustic networking.

Out of a myriad of network architectures, the distributed antenna system (DAS), which was originally proposed for the indoor wireless communications in 1980s [6], draws considerable attention in the wireless research community. Relative to the centralized antenna system (CAS), DAS which is formed by multiple distributed antenna elements (DAEs), provides a larger coverage and a higher throughput for the network with nomadic users [6]–[9].

In terms of the system outage performance, primary works on DAS in terrestrial radio environments can be found in [10]–[12]. Assuming a Rayleigh-lognormal flat fading channel, the outage performance of a single-cell single-use DAS is numerically studied in [10], which shows that the optimal performance can be achieved with fully distributed antennas. Using random matrix theory, a closed-form expression of the uplink outage probability in the distributed multi-input multi-output (MIMO) maximum-ratio-combining (MRC) system is derived in [11], with an assumption that the number of transmit antennas is no less than that of receive antennas. In [12], the downlink outage probability is derived for DAS in both single-cell and multicell environments with randomly located antennas and mobile users. It shows that in the multicell DAS, selection transmission is preferable to maximum ratio transmission, while the opposite is true for the single-cell DAS.

Relative to the DAS in terrestrial environments, studies on the underwater DAS are rare, despite the fact that the DAS structure has been used by engineers in underwater applications; two typical applications of the DAS are shown in Fig.1 where one is formed by the distributed nodes which are anchored at sea bottom and connected via cables, and the other is formed by the gateways which are geographically distributed at sea surface and can communicate via radio waves. The early testbed on the UWA networking can be found in [13] in the Atlantic underwater test and evaluation center (AUTEC), and an ocean technology testbed [14]. In this work, we consider the DAS in underwater environments. In particular, we investigate the outage performance of DAS during uplink transmissions in the frequency-selective UWA channel, and DASs with both single and multiple users are considered.

The UWA channel has been known different from the terrestrial radio channel in several aspects. Particularly, the path spreading-loss exponent in the terrestrial radio channel varies from 3 ~ 6, while in the UWA channel, the path spreading-loss exponent is practically taken as 1.5, which is much less than its terrestrial counterpart. Meanwhile, different from the terrestrial radio channel, the UWA channel also incurs signal absorption loss, which is highly frequency-dependent and proportional to the transmission distance.

Taking into account the propagation characteristics, the following observations for underwater DAS are obtained.

• In the single-user system, the DAS enjoys lower outage
probabilities than CAS in the moderate transmission SNR region, and the superiority of DAS over CAS increases as the signal frequency increases.

- In the multiuser DAS, a multicell based uplink transmission yields better outage performance than the single-cell based uplink transmission.

The rest of the paper is organized as follows. A system model for the multiuser underwater DAS is developed in Section II. The outage probabilities of the single-user DAS and multiuser DAS are derived in Sections III and IV, respectively. Conclusions are drawn in Section V.

II. SYSTEM MODEL FOR MULTIUSER UNDERWATER DAS

We consider an underwater distributed antenna system with $N_u$ users, and $N_r$ DAEs which are geographically distributed within an operation area, as shown in Fig. 2. To facilitate our analysis, we consider a circular operation area of radius $R_{op}$, and assume that users are uniformly distributed within the disk. Let $(\rho_{\mu}, \theta_{\mu})$ denote the polar coordinate of the $\mu$th user, which are independently distributed with the probability density function (PDF)

\[
p(\rho_{\mu}) = \frac{2\rho_{\mu}}{R_{op}^2}, \quad \rho_{\mu} \in [d_{\text{min}}, R_{op}],
\]

\[
p(\theta_{\mu}) = \frac{1}{2\pi}, \quad \theta_{\mu} \in [0, 2\pi]
\]

for $\mu = 1, \ldots, N_u$, where $0 < d_{\text{min}} \ll R_{op}$ is the minimum distance of each user to the DAE. Define $(D_{\nu,\mu}, \nu)$ as the Cartesian coordinate of the $\nu$th DAE. The distance between the $\mu$th user and the $\nu$th DAE is thus

\[
d_{\nu,\mu} = \sqrt{\rho_{\mu}^2 + D_{\nu,\mu}^2 - 2\rho_{\mu}D_{\nu,\mu}\cos(\theta_{\mu} - \theta_{\nu})}.
\]

We consider an underwater DAS system with a center frequency $f_c$ and bandwidth $B$. Let $s_{\mu}(f)$ denote the signal transmitted by the $\mu$th user in the frequency domain, and define $H_{\nu,\mu}(f)$ as the channel fading coefficient between the $\mu$th user and the $\nu$th DAE. To make the problem tractable, we assume an identical number of independent subchannels between each user and DAE pair, denoted by $N_{pa}$. Although in reality the numbers of paths in the channel of different user-base-station pairs could be very different, results obtained with this assumption shed insights into the outage performance of the underwater DAS. We denote $\{f_1, \ldots, f_{N_{pa}}\}$ as the center frequencies of the $N_{pa}$ consecutive subbands within $[f_c - B/2, f_c + B/2]$, and the bandwidth of each subband is $\Delta f = B/N_{pa}$.

The received signal of one particular frequency $f_p$ at the $\nu$th DAE is formulated as

\[
y_{\nu}(f_p) = \sum_{\mu=1}^{N_u} H_{\nu,\mu}(f_p)s_{\mu}(f_p)e^{-j2\pi f_p d_{\nu,\mu}} + w_{\nu}(f_p),
\]

for $\nu = 1, \ldots, N_r$, and $p = 1, \ldots, N_{pa}$, where $H_{\nu,\mu}(f_p)$ follows a complex Gaussian distribution $CN(0, \sigma^2(f_{\nu,\mu}, f_p))$ in which $\sigma^2(d_{\nu,\mu}, f)$ depicts the signal attenuation after transmitting a distance of $d_{\nu,\mu}$, and $w_{\nu}(f_p)$ is the ambient noise following a complex Gaussian distribution $CN(0, N_0)$. In UWA channels, the signal attenuates according to

\[
\sigma^2(d_{\nu,\mu}, f) \propto e^{-\alpha(f)d_{\nu,\mu}} d_{\nu,\mu}^{-\beta}
\]

where $\alpha(f)$ and $\beta$ are the frequency-dependent absorption coefficient and the path spreading-loss exponent, respectively.

III. OUTAGE PERFORMANCE OF THE SINGLE-USER DAS

A. Outage Probability in Wideband Systems

In this section, we consider the outage performance of single-user DAS in frequency-selective channels. The system input-output relationship is expressed in (2) with $N_u = 1$. For notational convenience, we abbreviate $H_{\nu,1}(f_p)$ as $H_{\nu,p}$.

Define $\tilde{\gamma}$ as the transmission signal-to-noise ratio (SNR)

\[
\tilde{\gamma} := \frac{P}{B_0 N_0}
\]

where $P$ is the transmitted signal power. For the single-user DAS, based on the input-output relationship in (2), the outage probability is formulated as

\[
\Pr(\tilde{\gamma}, R | (\rho, \theta)) = \Pr \left[ \sum_{p=1}^{N_{pa}} \log_2 \left[ 1 + \tilde{\gamma} \sum_{\nu=1}^{N_r} |H_{\nu,p}|^2 \right] < R \right],
\]

where $R$ denotes the data rate with a unit of bits/sec/Hz.

Define

\[
\mathcal{I}_p = \log_2 \left[ 1 + \tilde{\gamma} \sum_{\nu=1}^{N_r} |H_{\nu,p}|^2 \right].
\]

For CAS, with the mathematical result in (20), the PDF of $\mathcal{I}_p$ is formulated as

\[
f_p(z \mid (\rho, \theta)) = \frac{2^z \ln 2}{\tilde{\gamma} \sigma^2_{\text{CAS},p}} \exp \left( -\frac{2^z - 1}{\tilde{\gamma} \sigma^2_{\text{CAS},p}} \right) \frac{2^z - 1}{\tilde{\gamma} \sigma^2_{\text{CAS},p}} \left[ \frac{1}{(N_r - 1)!} \right].
\]

For DAS, with the mathematical result in (21), $\mathcal{I}_p$ has a PDF formulated as

\[
f_p(z \mid (\rho, \theta)) = \sum_{i=1}^{N_r} 2^z \ln 2 \exp \left( -\frac{2^z - 1}{\tilde{\gamma} \sigma^2_{\text{CAS},p}} \right) \left[ \frac{2^z - 1}{\tilde{\gamma} \sigma^2_{\text{CAS},p}} \right] \left[ \frac{1}{(N_r - 1)!} \right].
\]

The PDF of $\sum_{p=1}^{N_{pa}} \mathcal{I}_p$ given $(\rho, \theta)$ therefore can be obtained as a convolution of $N_{pa}$ functions,

\[
f(z \mid (\rho, \theta)) = f_1(z \mid (\rho, \theta)) \ast \cdots \ast f_{N_{pa}}(z \mid (\rho, \theta)).
\]
We first consider a narrowband system with $N_{pa} = 1$. For $R_{op} = 4$ km and $R = 1$ bit/sec, the outage probabilities of DAS and CAS at different operation frequencies, are shown in Fig. 4. Two observations are in order.

- The performance gap between DAS and CAS in the moderate transmission SNR region increases as the system frequency increases;
- In the high transmission SNR regime, there exists a crossing point of the outage probabilities of CAS and DAS, meaning that CAS can catch up and surpass DAS when the transmission power is sufficiently large. The outage probability at the crossing point decreases as the frequency increases.

2) Wideband systems: Fig. 5 demonstrates the outage probabilities of the wideband systems with different operation area sizes and different number of DAEs, respectively. The advantage of DAS over CAS is observed to be pronounced as the size of the operation area and the number of DAEs increase. Meanwhile, relative to the narrowband system, the crossing of the outage probabilities of the wideband DAS and CAS happens at a lower outage probability, which makes DAS a more appealing choice than CAS in the practical outage probability range.

IV. OUTAGE PERFORMANCE OF THE MULTIUSER DAS

A. Outage Probability in Wideband Systems

In a multiuser DAS, due to the low propagation speed of acoustic waves in water, signals from different users usually have different time-of-arrivals; the signals aligned at one DAE could have a very large time-difference-of-arrivals at other DAEs. Given the asynchronism of signals from multiuser at DAEs, we consider the outage probability of a low-complexity single-user decoding method.

Without loss of generality, we focus on the outage performance analysis of the first user. For notional convenience, we define $\rho := [\rho_1, \ldots, \rho_{N_a}]$, and $\theta := [\theta_1, \ldots, \theta_{N_a}]$. Based on (3), after the MRC of signals at all DAEs, the conditional outage probability of the first user is

$$p_{\text{out}}(\gamma, R) = \Pr \left[ \sum_{p=1}^{N_{pa}} \log_2 \left( 1 + \frac{\sum_{\mu=1}^{N_{a}} |H_{\nu,1,p}|^2}{\sum_{\mu=2}^{N_{a}} |H_{\nu,\mu,p}|^2 + 1/\gamma} \right) < R \right],$$

where $H_{\nu,\mu}(f_p)$ is abbreviated as $H_{\nu,\mu,p}$.

Define the signal-to-interference-plus-noise ratio (SINR) of signals from the first user at the $r$th DAE as

$$r_{\nu,p} := \frac{|H_{\nu,1,p}|^2}{\sum_{\mu=2}^{N_{a}} |H_{\nu,\mu,p}|^2 + 1/\gamma}.$$

Based on the mathematical result in (23), the cumulative distribution function (CDF) of $r_{\nu,p}$ is

$$F(r_{\nu,p} \mid (\rho, \theta)) = 1 - \exp \left( -\frac{r_{\nu,p}}{\gamma \sigma_{\nu,1,p}^2} \prod_{\mu=2}^{N_{a}} \frac{1}{r_{\nu,p}/\sigma_{\nu,1,p}^2 + 1/\sigma_{\nu,\mu,p}^2} \right).$$
with the PDF formulated as

\[ f(r_{\nu,p} \mid (\rho, \theta)) = \frac{1}{\bar{\gamma}_\nu} \left[ 1 + \frac{N_p}{\bar{\gamma}_\nu} + \sum_{\mu=2}^{N_p} \frac{1}{\bar{\gamma}_\nu + 1/\sigma_{\nu,\mu}^2} \right] \times \exp\left(-\frac{r_{\nu,p}}{\bar{\gamma}_\nu} \right) \prod_{\mu=2}^{N_p} \frac{1}{\bar{\gamma}_\nu + 1/\sigma_{\nu,\mu}^2}. \]  

(16)

After MRC, the PDF of \( r_p := \sum_{\nu=1}^{N_p} r_{\nu,p} \) is

\[ f_{r_p}(r_p \mid (\rho, \theta)) = f(r_{1,p} \mid (\rho, \theta)) \cdots f(r_{N_p,p} \mid (\rho, \theta)). \]  

(17)

The PDF of the instantaneous mutual information \( z_p := \log_2(1 + \bar{\gamma}_p) \) can be obtained as

\[ f_{z_p}(z_p \mid (\rho, \theta)) = 2^z \ln 2 f_{r_p}(r_p \mid (\rho, \theta)). \]  

(18)

Similar to (10), the PDF of the sum-rate \( z := \sum_{p=1}^{N_p} z_p \) is obtained as the convolution of all the PDFs of \( f_{z_p}(z_p \mid (\rho, \theta)) \),

\[ p_{\text{out}}(\bar{\gamma}, R) = \int_0^R \int_{\rho} \int_{\theta} f(z \mid (\rho, \theta)) p(\rho) p(\theta) d\rho d\theta. \]  

(19)

B. Numerical Results

In a large network, we consider two multiuser uplink transmission system structures. In the first structure, all the users are uniformly distributed within the overall operation area. In the second structure, we split the network into multiple cells, and each cell has one user which is uniformly distributed within the cell, as shown in Fig. 6. In numerical results, the radius of each circle is set as 4 km.

1) Outage probability in the single-cell structure: Taking the DAS in Fig. 6 as a single-cell, Fig. 7 demonstrates the outage probabilities of DAS and CAS with difference number of users at several system center frequencies. One can see that the outage probability increases as the number of users increases. At high frequencies (\( f_c \geq 10 \text{ kHz} \)), DAS outperforms CAS uniformly. Meanwhile, due to multiuser interference, systems...
at relatively low frequencies ($f_c \leq 1$ kHz) are observed having higher outage probabilities that the systems at medium frequencies (e.g., $f_c = 5$ kHz).

2) Single-cell versus multicell structure: Fig. 8 shows the outage probability of both the single-cell structure and the multicell structure. Due to the multiuser interference, the outage probability exhibits an error floor as the transmission power increases. Meanwhile, comparing the outage performance of the two DAS cellular structures with a large transmission power, a better outage performance can be achieved in multi-cell structure through reducing the level of multiuser interference, especially for systems at high frequencies.

V. CONCLUSIONS

In this work, we derived and compared the outage probabilities of DAS and CAS in the frequency-selective underwater acoustic channels. With a moderate transmission SNR, DAS was shown outperforming CAS in both single-user and multi-user uplink transmissions, and the performance gap increases as any of the system center frequency, the number of DAs, or the operation area size increases. For multiuser transmissions, an outage error floor was observed due to the multiuser interference. A multicell network structure can reduce the multiuser interference relative to the structure that takes the operation area as a single cell.

APPENDIX: MATHEMATICAL RESULTS

Result 1: For $n$ i.i.d exponential random variables $X_i$ with parameter $\chi$, define $Y = \sum_{i=1}^{n} X_i$. Using the Laplace transform, it can be shown that $Y$ follows a gamma distribution $(1, \chi)$, with the cumulative distribution function (cdf) formulated as

$$F_Y(y) = 1 - e^{-\chi y} \sum_{i=0}^{n-1} \frac{(\chi y)^i}{i!}, \quad (20)$$

Result 2: For $n$ independent exponentially distributed random variables $\{X_i, i = 1, \cdots, n\}$ with piecewise distinct parameters denoted by $\chi_1 < \chi_2 < \cdots < \chi_n$, respectively, the cdf of $Y = \sum_{i=1}^{n} X_i$ is

$$F_Y(y) = \sum_{i=1}^{n} \frac{1 - e^{-\chi_i y}}{\chi_i}, \quad (21)$$

Result 3: Consider $n$ independent exponentially distributed random variables with parameter of the $i$th variable $X_i$ denoted by $\chi_i$, and define $Y = \sum_{i=1}^{n} X_i$. Denote $X_0$ as an exponentially distributed random variable with parameter $\chi_0$. Define $Z := \frac{X_0}{Y + a}$, where $a$ as a positive constant. We have

$$\Pr(Z \leq z) = \Pr(X_0 \leq z(Y + a)) = \int \Pr(X_0 \leq z(y + a)) f_Y(y) dy. \quad (22)$$

The PDF $f_Y(y)$ can be computed with the result in (20) or (21). Given the exponential distribution of $X_0$, the CDF of $Z$ can be obtained as

$$F_Z(z) = 1 - e^{-\alpha z} \prod_{i=1}^{n} \frac{1}{\chi_i}. \quad (23)$$

For $\chi_i = \chi, \forall i = 1, \cdots, n$, we have

$$F_Z(z) = 1 - \frac{1}{(\chi z + \chi)^n}. \quad (24)$$

REFERENCES


