ABSTRACT
In this paper, we take a physical layer approach and investigate the outage probability in frequency selective underwater channels with uncoordinated multiple access and bursty transmissions. We first derive the statistical distribution of the interference as seen by a typical receiver, considering both burstiness of interference transmissions and spatial distributions of the interferers. We then derive expressions for the probability density functions of the frequency-dependent signal to noise and interference ratio. Expressions for the outage probability of a typical link between two nodes are obtained in the frequency selective scenarios and the flat fading special cases. The outage probabilities depend on the bursty transmission probability and the number of interfering transmitters. Simulation results are presented that validate the theoretical results and illustrate typical features of the interference in an underwater network.

Categories and Subject Descriptors
C.2.1 [Network Architecture and Design]: Network communications; H.4 [Information System Application]: Communications Applications

General Terms
Underwater acoustic communications, performance analysis

Keywords
Outage probability, bursty transmission, random access network, interference distribution

1. INTRODUCTION
Underwater wireless networking (UWN) plays an important role in underwater exploration, data gathering, command and control. Establishing communication networks in underwater environment is challenging due to energy absorption by water, long propagation delays and dispersion, fast channel variation and Doppler effects, and interference from natural and man-made acoustic interferences.

Depending on the underwater communication needs, in many cases the communications are bursty in nature — good throughput is needed for short durations. Such bursty communications are usually accommodated with media access control (MAC) layer protocol design. Interference when detected is avoided with back-off mechanism. Examples include for example the Aloha and carrier sense multiple access (CSMA), e.g. [1, 2]. Such schemes may work well for terrestrial wireless networks and wired networks, where signals propagate at speed of light and the round trip delay is usually very small compared to the interval of transmitting a single packet. For underwater communications, however, the round trip delay can be in the order of several seconds for a communication link between two nodes that are a few kilometers apart. Simply relying on upper layer (higher than physical layer) protocols to resolve the issue of interference may not be very efficient.

From the physical layer perspective, outage performance of terrestrial communications has been characterized in previous works. Based on power loss laws, outage probability in non-fading channel was derived in narrow band multi-hop packet radio networks in [8]. The effect of fading in ad hoc networks was considered in terms of outage probability in [11]. Lower and upper bounds on outage probability were derived in [10] for various channel distributions in terrestrial communications. The temporal variations of ad hoc networks were discussed in [6], where the time-varying channel was modeled by a finite Markov chain. The randomness of transmitting packets in both spatial and time domain was studied through MAC layer protocol design in [5]. In underwater communications, based on the probability of a successful transition, the network throughput density was analyzed in terms of transmitter density and the operating frequency in [9], and the results were verified by deployment-dependent network. The stochastic behavior of random traffic arrivals was modelled in [4] according to transmission protocols.

In this paper, we investigate the issue of interference in underwater communication networks from a physical layer perspective. Specifically, we assume that the multiple transmissions are not coordinated, and seek to answer the following questions: 1) To what extent reliable communication between two nodes is possible in the presence of bursty interferences; and 2) What is the outage behavior of a typical communication link under the assumption of uncoordinated bursty transmissions and realistic frequency selective propagation environment. The distribution of
aggregate interference is derived for underwater channels, for terrestrial channels, and for large networks. Then, the probability density functions (PDF) of signal to interference and noise ratio (SINR) in highly frequency-dependent channel are presented. Outage probabilities are derived in both flat fading and frequency selective fading channels. Simulation results validate the theoretical interference distribution and illustrate the impact of the probability of bursty transmissions on outage probability in acoustic underwater networks with random access.

2. SYSTEM MODEL

2.1 Network model

Consider a distributed UWN system consisting of \( N + 2 \) nodes in the system, numbered from 0 to \( N + 1 \). Node \( N + 1 \) intends to transmit a message to node 0, and node 1 to \( N \) are potential interferers. At any time instant, each node from 1 to \( N + 1 \) transmits independently with probability \( p \), and remains silent with probability \( 1 - p \). We assume that the transmit power spectral density of all nodes when transmitting is \( P(v) \).

2.2 Channel model

The channel between a transmitter and a receiver that are distance \( d \) apart is modeled as frequency selective with frequency response at frequency \( \nu \) denoted as \( H(d, \nu) \). The channel gain \( H(d, \nu) \) is modeled as complex Gaussian, zero-mean, and its variance is given by \([7]\):

\[
\sigma^2(d, \nu) = d^{-\alpha}a(\nu)^{-d} \tag{1}
\]

where \( \alpha \) is the path loss exponent (spreading factor) which depends on the geometry of propagation environment, and \( a(\nu) \) models part of the channel gain that depends on the distance exponentially due to absorption, and the exponent is frequency dependent. Typical values for \( \alpha \) are 1, 1.5, 2 for cylindrical, practical and spherical spreading, respectively.

We assume that the channels are quasi-static so that within a short duration, they can be modelled as Linear and Time-Invariant (LTI). Let \( x_j(\nu) \) denote the transmitted signal from the \( j \)th node at frequency \( \nu \). If a node is not transmitting, the transmitted signal is set to zero. The received signal at frequency \( \nu \) can be written as

\[
y_0(\nu) = \sum_{j=1}^{N+1} H_{0,j}(d_{0,j}, \nu)x_j(\nu) + n(\nu) \tag{2}
\]

where \( H_{0,j}(d_{0,j}, \nu) \) denotes the channel gain from \( j \)th node to node 0, and \( n(\nu) \) models the additive noise. The additive noise is independent of all transmitted signals, zero mean, complex Gaussian distributed, and spectrally white with power spectral density \( N_0 \).

3. DISTRIBUTION OF INTERFERENCE

Due to the asynchronous transmissions, the interference present at node 0 is time-varying and frequency dependent. In order to characterize the achievable rate from node \( N + 1 \) to 0, and outage probability, we first characterize the statistical distribution of interference signal. Because of the frequency dependency of the interference signal, we will analyze the distribution of the interference as a function of frequency.

3.1 PDF of network interference

The number of interferers seen by node 0 at any time is binomial distributed, namely the probability that \( k \) out \( N \) interference signals are present in the received signal at node 0 is

\[
p(k) = \binom{N}{k} p^k (1 - p)^{N-k}. \tag{3}
\]

Let \( I_{\nu,k} \) denote the total interference at frequency \( \nu \) when there are \( k \) interferers, which is a random variable due to i) random locations of the interfering nodes; and ii) channel fading. The PDF of the interference \( I_{\nu} \) seen by node 0 at frequency \( \nu \) can be written as

\[
f_{I_{\nu}}(x) = \sum_{k=0}^{N} \binom{N}{k} p^k (1 - p)^{N-k} f_{I_{\nu,k}}(x), \tag{4}
\]

where \( f_{I_{\nu,k}}(x) \) is the PDF of \( I_{\nu,k} \).

Thanks to the independence of the interference signals, \( I_{\nu,k} \) is the sum of the \( k \) interference powers. Because all the interference nodes are randomly located, the PDF of \( I_{\nu,k} \) is the \( k \)-fold convolution of \( I_{\nu,1} \):

\[
f_{I_{\nu,k}}(x) = f_{I_{\nu,1}}(x) \otimes f_{I_{\nu,1}}(x) \otimes \ldots \otimes f_{I_{\nu,1}}(x). \tag{5}
\]

3.2 Interference from one node

We next derive the distribution of the interference signal \( I_{\nu} \) from a single interferer, assuming that the interferer is uniformly distributed in the given area. Without loss of generality, suppose node 1 is the interferer. Let \( P(\nu) \) denote the power spectral density (PSD) of the transmitted signal. The signal transmitted by node 1 will go through the channel \( H_{0,1}(d_{0,1}, \nu) \) and arrive at the receiver node 0. Let \( Z_1 \) denote the location of node 1, within a certain area \( \mathcal{A} \). Since \( d_{0,1} \) depends on the location \( Z_1 \), the interference created, conditioned on the location \( Z_1 \) of the node 1 is

\[
I_{\nu,1} = |H_{0,1}(d_{0,1}, \nu)|^2 P(\nu). \tag{6}
\]

Due to the complex Gaussian channel fading assumption (cf. (1)), the interference \( I_{\nu,1} \) is exponentially distributed with mean \( d_{0,1}^{-\alpha}a(\nu)^{-d} P(\nu) \). Let \( f_{I_{\nu,1}}(x|z_1) \) denote this distribution. The PDF of \( I_{\nu,1} \), averaged over the locations of \( Z_1 \) is

\[
f_{I_{\nu,1}}(x) = \int_{z_1 \in \mathcal{A}} f_{I_{\nu,1}}(x|z_1)f_{Z_1}(z_1)dz_1 \tag{7}
\]

where \( f_{Z_1}(z_1) \) is the PDF of the location \( Z_1 \).

3.3 MGF of network interference

Because of the convolution involved in the calculation of \( f_{I_{\nu,k}}(x), \) cf. (5), it is easier to express the moment generating function (MGF) of the interference signal. Let \( M_{I_{\nu,1}}(s) \) denote the MGF of \( I_{\nu,1} \):

\[
M_{I_{\nu,1}}(s) = \mathbb{E} [e^{sI_{\nu,1}}]. \tag{8}
\]

The MGF of \( I_{\nu,k} \) is therefore \( M_{I_{\nu,1}}^k(s) \). It follows that the MGF of \( I_{\nu} \) is

\[
M_{I_{\nu}}(s) = \sum_{k=0}^{N} \binom{N}{k} p^k (1 - p)^{N-k} M_{I_{\nu,1}}^k(s) \nonumber
\]

\[
= [1 - p + pM_{I_{\nu,1}}(s)]^N. \tag{9}
\]
Having derived the MGF of the total interference, we can obtain the frequency dependent interference by doing the inverse Laplace transform of $M_{I_0}(-s)$.

4. DISCUSSION

4.1 The discrete component in the PDF

With reference to (4), there is a probability of $(1-p)^N$ that no interferer is transmitting, in which case the interference signal power is zero. Therefore, the PDF of the interference when\[ \text{PDF of } N_p \]

This term may be negligible when $N_p$ is large, but is significant when $N_p$ is small. When at least one interferer is present, and fading is also present, then the interference signal is continuously distributed at any frequency.

4.2 Terrestrial vs underwater channels

The interference distribution varies according to the path loss law. We next look at two cases of path loss: the underwater channel and the terrestrial communication channels. We assume the field is a circular area of radius $R$, where the reference node 0 is placed at the center of the region. Under this assumption and the assumption that all interfering nodes are uniformly distributed within the area, the PDF of the distance $d_{0,j}$ between any interfering node $j$ and the receiver 0 is $f_{d_{0,j}}(r) = \frac{2r}{R^2 - R_{th}^2} \delta(r)$, $R_{th} \leq r \leq R$, where $R_{th}$ is the minimum distance of an interferer from the node 0, which is a small constant.

4.2.1 Underwater Communications

An exponential distribution with mean $1/\lambda$ has MGF $\lambda/(\lambda - s)$. Using this result and the channel model (2), the MGF in (8) can be expressed as follows:

\[
M_{I_0,1}(s) = \int_{R_{th}}^{R} \frac{s\alpha(\nu)^{\nu}}{P(\nu) - s R^2 - R_{th}^2} dr \frac{2r}{R^2 - R_{th}^2} \delta(r) = 1 + \frac{2s}{R^2 - R_{th}^2} \int_{R_{th}}^{R} \frac{r}{s\alpha(\nu)^{\nu}/P(\nu) - s} dr.
\]

4.2.2 Terrestrial Communications

In terrestrial communications the assumption of small absorption losses is usually assumed zero. This corresponds to setting $s\alpha(\nu) = 1, \forall \nu$ in (1). The $M_{I_0,1}(s)$ can be computed as the closed form expression given in (3). In particular, if $\alpha = 2$, it is the model of free space propagation. In this case, it is possible to obtain the MGF $M_{I_0,1}(s)$ explicitly:

\[
M_{I_0,1}(s) = 1 + \frac{\sqrt{sP(\nu)}}{2(R^2 - R_{th}^2) \ln (\frac{R^2 - \sqrt{sP(\nu)}}{R_{th}^2 - \sqrt{sP(\nu)}})},
\]

where $\Re(s) < R_{th}^2/P(\nu)$. If $\alpha = 4$, which is the case for a two-ray channel [2], the MGF is

\[
M_{I_0,1}(s) = 1 + \frac{\sqrt{sP(\nu)}}{2R_{th}} \ln (\frac{R_{th}^2 - \sqrt{sP(\nu)}}{R_{th}^2 - \sqrt{sP(\nu)}}).
\]

4.3 Large networks

If we set $N_p$ to be a fixed constant $\bar{\lambda}$, and let $N \to \infty$, we have $p \to 0$. The parameter $\bar{\lambda}$ represents the average number of interfering nodes. Making use of the calculus fact

\[
\lim_{N \to \infty} \left(1 + \frac{x}{N}\right)^N = e^x,
\]

we obtain from (9) that

\[
\lim_{N \to \infty} M_{I_0,1}(s) = e^{\bar{\lambda}(M_{I_0,1}(s)-1)}.
\]

If there is no signal attenuation, no frequency selectivity, and no fading, then the MGF of one interferer is $M_{I_0,1}(s) = e^{P_{b}(\nu)s}$ and the MGF of total interference $I_0$ is $e^{\bar{\lambda}(P_{b}(\nu)s-1)}$, which is the MGF of a Poisson distribution. So our result includes the Poisson interference distribution as a special case.

5. OUTAGE PROBABILITY

In this section, we consider the outage probability of a typical communication link in a distributed random access network with frequency selective channels. Let $d_{0,N+1}$ be the distance between the transmitting node ($N+1$) and the intended receiver node 0. Let $\gamma(d_{0,N+1}, \nu)$ denote SINR at frequency $\nu$ as measured by node 0. The mutual information between the transmitted signal from the ($N+1$)-th node and received signal at node 0, over the frequency band $f_c - B/2$ and $f_c + B/2$, is as follows:

\[
I(d_{0,N+1}, f_c, B) = \int_{f_c-B/2}^{f_c+B/2} \log_2 (1 + \gamma(d_{0,N+1}, \nu)) d\nu.
\]

The received SINR of node 0 at frequency $\nu$ can be expressed as

\[
\gamma(d_{0,N+1}, \nu) = \frac{P(\nu) H_{0,N+1}(d_{0,N+1}, \nu)^2}{N_0 + I_0} = \frac{H_{0,N+1}(d_{0,N+1}, \nu)^2}{1/\gamma(\nu) + I_0/P(\nu)}
\]

where $\gamma(\nu) = P(\nu)/N_0$ is the transmitted signal to noise ratio at frequency $\nu$.

5.1 Frequency-dependent SINR

To analyze the system mutual information and outage probability, we will need the statistical distribution of the frequency-dependent SINR, which we derive in the following.

For a given $d_{0,N+1}$, $[H_{0,N+1}(d_{0,N+1}, \nu)]^2$ at node 0 is exponentially distributed. The distribution of total interference $I_0$ has also been derived in (4). Given these two distributions, we can write the cumulative distribution function (CDF) of SINR as

\[
\Pr \left[ \gamma(d_{0,N+1}, \nu) \leq z \right] = \int_0^z \Pr \left[ [H_{0,N+1}(d_{0,N+1}, \nu)]^2 \leq \frac{1}{\gamma(\nu) + I_0/P(\nu)} \right] df_{I_0}(x) dx
\]

\[
= \int_0^z \Pr \left[ [H_{0,N+1}(d_{0,N+1}, \nu)]^2 \leq \frac{1}{\gamma(\nu) + I_0/P(\nu)} \right] f_{I_0}(x) dx
\]

\[
= 1 - e^{-\lambda(d_{0,N+1}, \nu) z/(\bar{\lambda} P(\nu))} M_{I_0}(x) = 1 - e^{-\lambda(d_{0,N+1}, \nu) z/(\bar{\lambda} P(\nu))} M_{I_0,1}\left(\frac{\lambda(d_{0,N+1}, \nu) z}{P(\nu)}\right)
\]

where $\lambda(d_{0,N+1}, \nu) = d_{0,N+1}^2 a(\nu) d_{0,N+1}$.
5.2 Frequency flat channels

Based on the discussion leading to (14), we know that when $N$ is large, the outage probability for arbitrary path loss model under bursty transmission is approximately determined by the $\lambda = Np$. If the overall channel is frequency flat, then the SINR is not dependent on the frequency. The outage probability in this case can be denoted by $q(\lambda(d_0,N+1),\beta(d_0,N+1))$, where $\lambda(d_0,N+1) = d_0^2 N+1$, and $\beta$ is the SINR threshold for defining the outage event.

Additionally, it is worthy noting that for terrestrial communications, it is possible to obtain closed-form expressions for PDFs and CDFs of the SINR. Particularly, if i) the noise is negligible, ii) the transmissions are always on, namely $p = 1$; iii) channels are frequency-flat, namely $a(\nu) = 1$; iv) the network is infinitely large, namely $R \to \infty$; and v) $\alpha = 4$, then the outage probability can be obtained as

$$q(\lambda(d_0,N+1),\beta(d_0,N+1)) = 1 - M_{\nu}(\frac{\lambda(d_0,N+1)z}{P(\nu)})$$

$$= 1 - (1 - \frac{\pi \sqrt{\lambda(d_0,N+1)z}}{2R^2})^N.$$  (18)

Making use of (13), we have

$$q(\lambda(d_0,N+1),\beta(d_0,N+1)) = 1 - \exp(-\frac{N}{\pi R^2 \pi d_0^2 N+1 \sqrt{\beta \pi}})$$

which recovers the result in [10, eq. 61], with transmission intensity equal to $\lambda = N/\pi R^2$ per m². If we consider the randomness of node $0$'s location, the overall outage probability in flat fading channel can be computed as

$$P_{out}(\beta) = \int_{R_{th}}^R q(\lambda(d_0,N+1),\beta(d_0,N+1)) f_{d_0,N+1}(r) dr.$$  (19)

5.3 Frequency selective channels

In wideband frequency selective fading channel, assuming that the transmitter is sending information at rate $R_{op}$ bits per second, the outage probability, conditioned on $d_0, N+1$ is

$$P_{out}^{(W)}(d_0,N+1, f_\nu) = \Pr [I(d_0,N+1, f_\nu) < R_{op}].$$  (20)

Let $d_0 = [d_0,1, \ldots, d_0,j, \ldots, d_0,k]$ denote the locations of $k$ interferers. Given these interferer positions, the SINRs over the different frequencies are independently distributed because of the independence of $H_{0,j}(d_0,j, \nu)$. The SINR at frequency $\nu$ is as follows:

$$\gamma_k(d_0,N+1, \nu) = \frac{|H_{0,j}(d_0,N+1, \nu)|^2}{1/\gamma(\nu) + I_{0,k}/P(\nu)}.$$  (21)

Note that if we do not consider the SINR on the numbers and locations of the interferers, then the SINRs at different frequencies are statistically dependent random variables. Following (17), conditioned on the number $k$ of interferers and their locations $d_0$, the CDF of the SINR at frequency $\nu$, $\gamma_k(d_0,N+1, \nu)$, is

$$\Pr [\gamma_k(d_0,N+1, \nu) \leq z|d_0, k]$$

$$= 1 - e^{-\frac{\lambda(d_0,N+1, \nu)z}{P(\nu)}} \sum_{j=1}^k M_{\nu,j}(\frac{\lambda(d_0,N+1, \nu)z}{P(\nu)})$$

$$= 1 - e^{-\frac{\lambda(d_0,N+1, \nu)z}{P(\nu)}} \sum_{j=1}^k \lambda(d_0,j, \nu) + \lambda(d_0,N+1, \nu)z/P(\nu).$$

![Figure 1: The interference magnitude distribution](image)

Taking the derivative of the CDF with respect to $z$, we can obtain the PDF of $\gamma_k(d_0,N+1, \nu)$ conditioned on $d_0$ and $k$.

As an approximation to the mutual information (15), the whole frequency band can be divided into $M$ subbands, with spacing $\Delta\nu$. Then, the mutual information can be approximated by

$$I(d_0,N+1, f_\nu, B) = \sum_{m=1}^M \Delta\nu \log_2(1 + \gamma(m)(d_0,N+1)).$$  (22)

Denoting the mutual information at the $m$-th frequency interval $\Delta\nu$ as $Y_k^{(m)} = \Delta\nu \log_2(1 + \gamma(m))$, we can get the PDF of $Y_k^{(m)}$ conditioned on $d_0, k$, i.e.,

$$f_{Y_k^{(m)}}(y_k^{(m)}|d_0, k) = \frac{\ln 2}{\Delta\nu} f_{\gamma(m)}(y_k^{(m)}|d_0, k)(2^\frac{y_k^{(m)}}{\Delta\nu} - 1|d_0, k).$$

The sum-rate over bandwidth $B$ is $Y_k = \sum_{m=1}^M Y_k^{(m)}$. The PDF for the sum-rate can be computed as the convolution of all individual PDFs which are $f_{Y_k^{(m)}}(y_k^{(m)}|d_0, k)$. $m = 1, \ldots, M$. According to the law of total probability, the PDF of sum-rate $Y$ conditioned on $d$ is

$$f_{Y|d}(y|d) = \frac{N}{k=0} p(k) f_{Y_k}(y_k|d_0, k)$$  (23)

where $d^{1 \times N}$ represents the all locations of interfering nodes for convenience of expression. The overall outage probability in wideband frequency selective fading channel can be calculated through integrating $f_{Y|d}(y|d)$ over $d$. However, in this case, multi-fold integral is involved and exact evaluation may not be feasible when $N$ is large. Design of multiple access scheme for ad hoc network over frequency selective channels is a topic of future research.

6. SIMULATION RESULTS

We validate through simulations the theory for the interference distribution, PDF of received SINR, and outage probabilities in both flat and frequency selective fading channels. A reference receiver is placed at the center of a disk of radius $R$. For wideband UWN, autonomous underwater vehicles (AUVs) transmit the signals to their intended receivers in the band 1-20kHz. The interfering nodes are uniformly distributed in the network, and assumed homogeneous with starting times modelled as Poisson point
process with a mean of 0.05 packets per second. Each packet lasts 100 ms. Their signals propagate through underwater environment according to the frequency-dependent model as in (1), where $a$ is chosen as 1.5. Fig. 1 shows the interference magnitude distribution which contains a probability mass at zero and has a heavy tail. These two parts characterize the properties of bursty transmission communications, i.e., sparsity and possible interference with high power. Also, Fig. 1 illustrates the consistency between the theoretical analysis in (9) and numerical results.

The PDF of received SINR at reference node 0 is calculated in Fig. 2. Since the path loss is highly dependent on distance, especially the exponential term in (1), the transmitted power is attenuated significantly at the receiver. Meanwhile, the power is also strongly dependent on frequency, so the aggregated interference varies with frequency. The curves clearly show that as the frequency $\nu$ decreases, the average SINR increases.

Fig. 3 illustrates the outage probability at different $f_c$ versus transmitted signal-to-noise ratio. Depending on the transmission probability, it can be observed that the bursty transmission with smaller $p$ leads to smaller outage probability, as expected. The outage probability is also carrier frequency dependent. In terms of the total rate delivered, there will be an optimal value of the transmission probability $p$. The optimization of such parameters will be a topic of future research.

7. CONCLUSIONS

We investigated underwater communications with random access and derived the interference distributions for different propagation models. In-depth outage probability analysis was conducted for flat and frequency-selective channels. The discussed random access transmission mechanism incorporates both random traffic pattern and the random locations of the interfering nodes. Simulation results showed the impact of bursty transmission intensity on outage probability over wideband frequency selective underwater acoustic channel. Future research will include optimizing the transmission parameters and multiple accessing scheme design for frequency-selective wideband channels.

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8. REFERENCES